

# **Introduction to Reactor System**

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In this lecture, we shall discuss about the concepts of binding energy of a single nucleus and the average nuclear binding energy. This will be followed by discussions on neutron energies and various nuclear reactions.

At this end of this lecture, the learners will be able to

- (i) explain the origin of binding energy of a nucleus
- (ii) calculate the binding energy of a nucleus using Einstein's mass-energy relationship
- (iii) calculate the binding energy of a neutron
- (iv) distinguish between fast and slow neutron
- (v) identify various nuclear reactions
- (vi) list the applications of various nuclear reactions in reactors

## 1 Nucleus

We are aware that protons and neutrons comprise nucleus of an atom. Nucleus is positive, due to the presence of positively charged protons and the neutrons (no charge). Few elements have same atomic number 'Z' (same number of electrons) but possess different mass number 'A' (total sum of electrons and neutrons). Such elements are called *Isotopes*, the lightest of them being  ${}^2_1\text{D}$  (Deuterium) and  ${}^3_1\text{T}$  (Tritium) being the isotopes of  ${}^1_1\text{H}$  (Hydrogen) as shown in Figure 1.

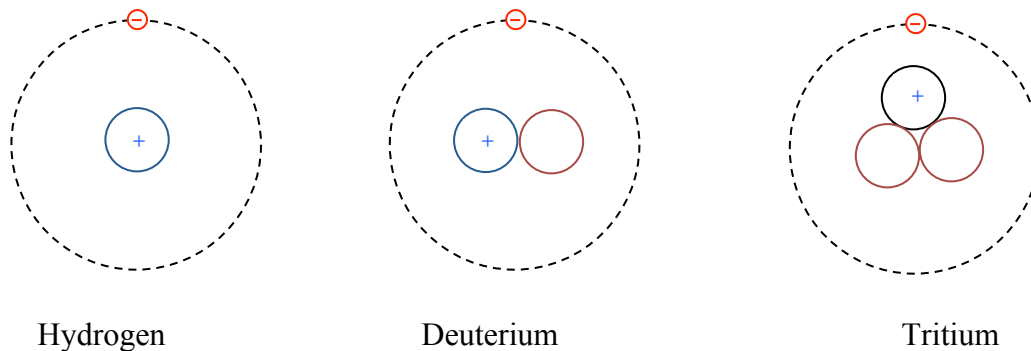


Fig 1. Isotopes of hydrogen

The difference between the atomic mass (A) and the atomic number (Z) is the neutron number (N). Hence, the isotopes of same element possess different neutron numbers (N). For the purpose of distinguishing isotopes, 'A' may be identified as the total number of **nucleons**, which are particles heavier than electron (proton and neutron are heavier than electron).

The size of a nucleus is much lesser than the size of an atom. Nucleus is denser than the atom, due to its smaller volume and larger mass of protons. The radius of a nucleus is related to the number of nucleons ( $A$ ) as follows:

$$R(\text{cm}) = 1.4 \times 10^{-13} A^{1/3}$$

The mass of proton, neutron, electron and atom are expressed in amu (atomic mass unit). One amu is equal to  $1.66 \times 10^{-24}$  g. Table 1 gives the mass of a neutron, proton, and an electron.

Table 1. Masses of neutron, proton and electron

Particle	Mass (amu)
Neutron	1.008665
Proton	1.007276
Electron	0.000549

### 1.1 Binding energy of a nucleus (BE)

As the nucleus contains protons (all of which are positively charged) and neutrons (no charge), one would expect electrostatic repulsion between protons to be high. Electrostatic repulsion between two charged particles varies inversely with the square of distance between them. Hence, as two protons come close to each other, one would expect electrostatic repulsion to make them move apart. However, the existence of nucleus testifies the fact that there exists a short-range attractive force stronger than the electrostatic repulsion. This short-range force is the *nuclear force* and is responsible for binding *nucleons* into a compact nucleus. A potential energy of binding (called binding energy) is associated with this force and hence energy equivalent to this must be supplied to disrupt a nucleus to component nucleons.

The binding energy, denoted as BE, of a nucleus may be calculated using Einstein's mass-energy relation as follows:

Let  $m_n$  be the mass of a neutron,  $m_H$  be the sum of mass of a proton and an electron, then the total mass of all constituent particles ( $m_{CP}$ ) of an atom with atomic number ( $Z$ ) and neutron number ( $N$ ) is given by

$$m_{CP} = Z(m_H) + N(m_n) \quad (1)$$

If mass of the atom is  $M$ , then the difference between the total mass of all constituent particles and that of the atom is the mass defect ( $m_d$ ), given by

$$m_d = m_{CP} - M = Z(m_H) + N(m_n) - M \quad (2)$$

Taking ‘ $m_H$ ’ and ‘ $m_n$ ’ as 1.007825 and 1.008665 amu, we get the *mass defect* as

$$m_d = Z(1.007825) + N(1.008665) - M \quad (3)$$

The unit of  $m_d$  is also amu.

Using Einstein’s relationship, one may relate energy and mass as  $E = mc^2$  (4)

Taking ‘ $m$ ’ as 1 amu,  $E = 1.494 \times 10^{-10} \text{ J} = 931 \text{ MeV}$  ( $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ )

For an atom with a mass defect ‘ $m_d$ ’, binding energy =  $m_d c^2$  (5)

From the tabulated values of Z, N and M for an atom the binding energy may be calculated using the equations (2) and (5)

*For data on the atomic masses refer Appendix – B, J. Kenneth Shultis, Richard E. Faw, “Fundamentals of Nuclear Science and Engineering”, 2002*

## 1.2 Average nuclear binding energies

Average nuclear binding energy =  $BE/A$  = Binding energy of a nucleus/number of nucleons

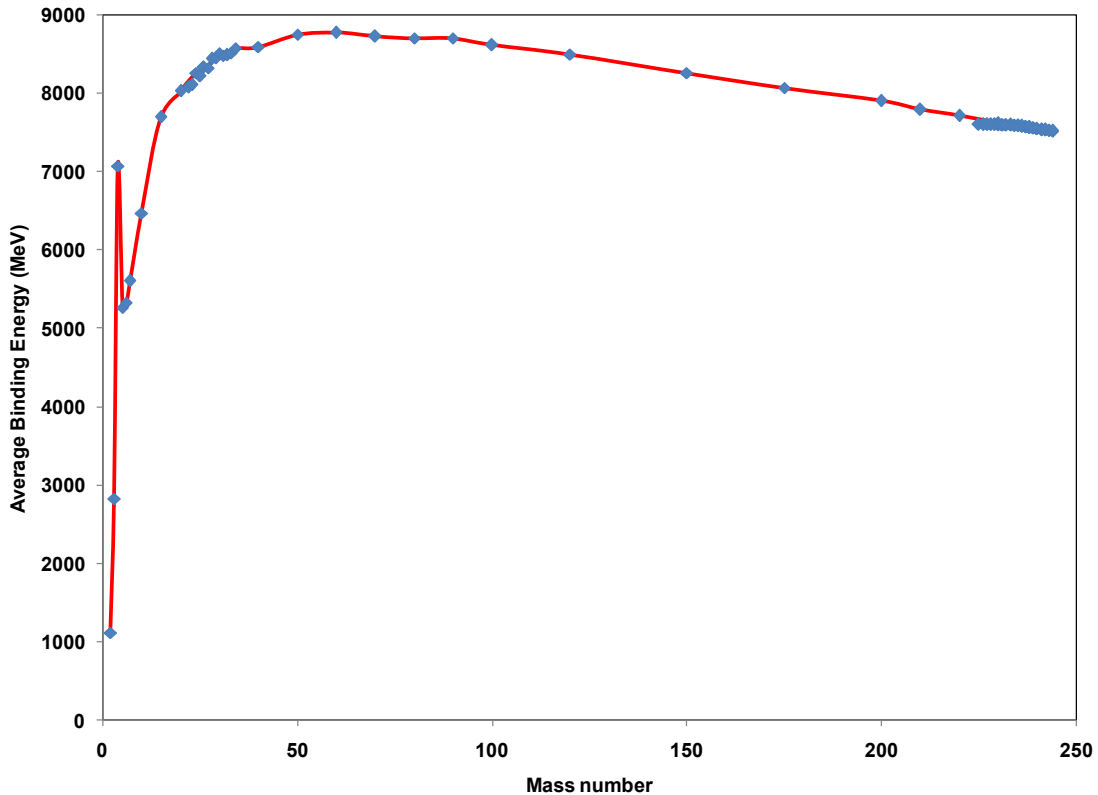


Fig 2. Average binding energy as a function of mass number

The elements with higher binding energy per nucleon are difficult to break up. Iron with a mass number of 56 has the highest binding energy per nucleon (average binding energy). Among the light elements Helium with a mass number of 4 has the highest average binding energy.

For data on atomic masses and average binding energies of nucleus of various isotopes, one may refer to “*LBNL Isotopes Project Nuclear Data Dissemination Home Page*”

**Example -1: Determine the binding energy of the nucleus in  $^{235}_{92}\text{U}$ . The mass of  $^{235}\text{U}$  atom is 235.0439231 amu.**

Recalling Eq. (2),  $m_d = Z(1.007825) + N(1.008665) - M$

$Z = 92$ ;  $A = 235$ ;  $N = A - Z = 143$ ,  $M = 235.0439231$  amu

Substituting above in Eq. (2) yields a mass defect ( $m_d$ ) of 1.9151 amu.

Recalling that 1 amu corresponds to 931 MeV, the binding energy of U-235 is  $1.915 \times 931 \sim 1793$  MeV

**Example – 2: Determine the average binding energy of the nucleus in  $^{239}_{94}\text{Pu}$ . The mass of  $^{239}\text{Pu}$  is 239.052163 amu.**

Recalling Eq. (2),  $m_d = Z(1.007825) + N(1.008665) - M$

$Z = 94$ ;  $A = 239$ ;  $N = A - Z = 145$ ,  $M = 239.052163$  amu

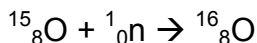
Substituting above in Eq. (2) yields a mass defect ( $m_d$ ) of 1.9398 amu.

Recalling that 1 amu corresponds to 931 MeV, the binding energy of U-235 is  $1.9398 \times 931 \sim 1805.965$  MeV

Average binding energy = binding energy/number of nucleons =  $1805.965/239 = 7.556$  MeV

### **1.3 Binding energy of a neutron**

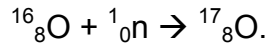
Similar to the calculation of binding energy of a nucleus, binding energy associated with addition of a neutron ( $B_n$ ) of the following type of neutron reaction can be calculated.



$$m_{\text{dnu}} = M_1 + m_n - M_2 \quad (6)$$

In Eq. (5) 'M<sub>1</sub>' and 'M<sub>2</sub>' are the mass of atom before (<sup>15</sup><sub>8</sub>O) and after neutron (<sup>16</sup><sub>8</sub>O) absorption. 'm<sub>dnu</sub>' is the mass defect of a neutron.

**Example - 3: Determine the binding energy of the neutron added during the following reaction:**



Recall Eq. (5)  $m_{\text{dnu}} = M_1 + m_n - M_2$

Mass of <sup>16</sup><sub>8</sub>O = M<sub>1</sub> = 15.9949146; Mass of <sup>17</sup><sub>8</sub>O = M<sub>2</sub> = 16.9991317;

m<sub>n</sub> = 1.008665

Substituting above in Eq. (5) yields,

m<sub>dnu</sub> = 0.004448 amu

Binding energy = m<sub>dnu</sub>c<sup>2</sup> = 4.141 MeV

### 1.4 Neutron Classification

Depending on the kinetic energy of a neutron, it may be classified into number of categories chief among which are thermal and fast neutrons

Neutrons with kinetic energies of about 0.025 eV are called thermal electrons, while those with kinetic energies around 1 MeV.

Recalling the relationship between speed and kinetic energy, speed of a neutron (u<sub>n</sub>) corresponding to a kinetic energy (E<sub>n</sub>) can be calculated using the following equation:

$$0.5m_n u_n^2 = E_n \quad (7)$$

From Eq. (7) At 20 °C, the speeds of thermal and fast neutrons are 2.2 km/s and 14000 km/s respectively.

## 2 Nuclear Reactions

These are the reactions involving the participation of atomic nuclei. Some of these reactions are spontaneous (Eg. Radioactivity) while few others are result of bombardment of nuclei with an energetic particle or radiation.

There are certain similarities and differences between nuclear and chemical reactions. The similarity lies in the fact that both these type of reactions obey the conservation laws: conservation of mass, energy, particles and the charge.

The difference lies in the amount of energy released. Nuclear reactions are highly energetic compared to chemical reactions. For example, fission of an atom of Uranium-235 releases about 210 MeV of energy, while energy released due to the formation of one molecule of CO<sub>2</sub> from the combustion of carbon is about 4.08 eV.

The types of nuclear reactions that are of importance in nuclear reactors are:

- (i) Elastic scattering
- (ii) Inelastic scattering
- (iii) Neutron capture
- (iv) Fission

### 2.1 Elastic scattering

This is a type of neutron (nuclear) reaction involving elastic collision of a neutron with a nucleus. The structure or mass of nucleus does not change as a result of this reaction. However, the speed of the neutron and its direction changes. As the name 'elastic' scattering suggests there is no change in kinetic energy of the system during this event.

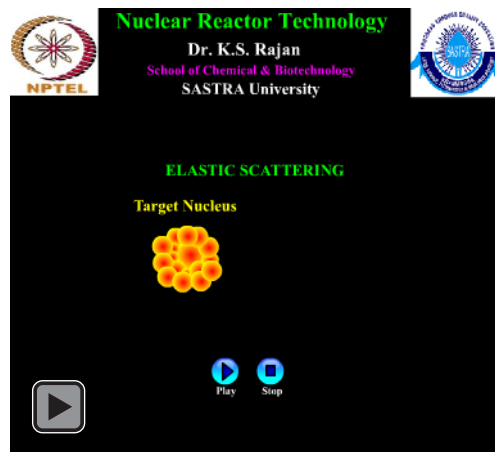


Fig 3. Animation of Elastic Scattering  
Note: Can be viewed only in Acrobat Reader 9.0 and above

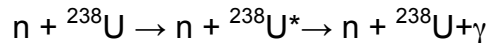


In nutshell, Elastic scattering is change in speed and direction of a neutron (**scattering**) without any change in total kinetic energy of the system (**elastic**), though the neutron loses a part of its kinetic energy and *slows down*.

The elastic scattering utilized in the nuclear reactors is the scattering of neutron by reaction with light nuclei (hydrogen, deuterium, carbon) resulting in a large loss of kinetic energy of neutron, without any loss in the number of neutrons. This is called '**moderation**' and the substances containing light nuclei (water, heavy water, graphite) are called '**moderators**'.

## 2.2 Inelastic scattering

This is a type of neutron (nuclear) reaction involving inelastic collision of a neutron with a nucleus. The nucleus reaches an excited state upon neutron irradiation, which decays rapidly to the ground state. This is accompanied by the emission of gamma radiation.



' $\gamma$ ' represents gamma radiation and  ${}^{238}\text{U}$  is the target nucleus

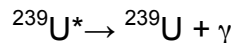
For inelastic scattering to occur, the energy of the neutron incident on a nucleus must necessarily be greater than the excitation energy of the lowest excited state of the nucleus. The initial slowing down of neutron in a reactor system is due to inelastic scattering.

## 2.3 Neutron capture

Different from elastic and inelastic scattering where there is no change in the atomic mass of target nucleus, neutron capture results in the increase in mass of target nucleus which reaches the excited state. Let us consider the following example



The excited nucleus then decays rapidly (within  $10^{-13}$  seconds) by emission of one or gamma radiation as follows:



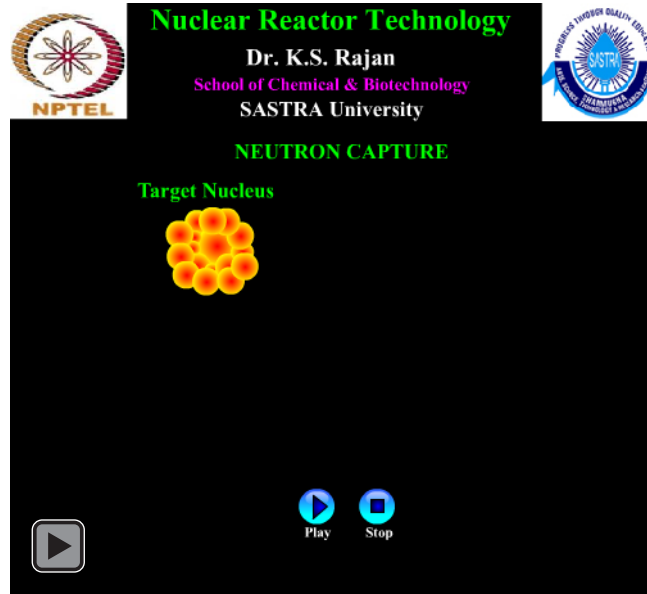


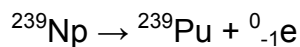
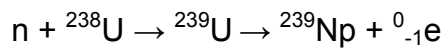
Fig 4. Animation of neutron capture  
 Note: Can be viewed only in Acrobat Reader 9.0 and above

The importance of neutron capture in a nuclear reactor is as follows:

- (i) Neutrons are captured by control rods which otherwise might initiate fission



- (ii) Transformation of one nucleus to the other is due to neutron capture, enabling production of isotopes of interest



The half-lives of  ${}^{239}\text{U}$  and  ${}^{239}\text{Np}$  are 23.5 min and 2.355 days respectively, shorter than the time involved in the production of fissile material  ${}^{239}\text{Pu}$ . The decay of  ${}^{239}\text{Pu}$ , an isotope with a high half-life of about  $2.41 \times 10^4$  years, is not appreciable in the reactor. The isotope that undergoes nuclear transmutation to produce a fissile isotope is called fertile isotope. In the above example  ${}^{238}\text{U}$  is the fertile isotope, which upon nuclear transmutation produces  ${}^{239}\text{Pu}$ .

### **3 References/Additional Reading**

1. “Fundamentals of Nuclear Science and Engineering”, J. Kenneth Shultis, Richard E. Faw, Marcel Dekker, 2002 (Chapter 4)
2. <http://ie.ibl.gov/toi.html>